Period 3, Sept 6, 2024

2. Find the following limits without a calculator. Show your work.

a) $\lim_{x\to-\infty} \frac{\sqrt{12x^2-4}}{x+2} = \lim_{x\to-\infty} \frac{\sqrt{12x^2}}{x} = \frac{\sqrt{12}\cdot\sqrt{x^2}}{x} = -\sqrt{12}$

$\lim_{x\to-\infty} \frac{|x|\sqrt{12}}{x} = \frac{|-50{,}000{,}000{,}000{,}000|\sqrt{12}}{-50{,}000{,}000{,}000{,}000}$

$-1\cdot\sqrt{12} = -\sqrt{12}$

b) $\lim_{x\to-\infty} \frac{5x^3-2x}{7+2x^4}$

c) $\lim_{x\to\infty} \frac{3e^x}{500x^{400}} = \infty$

Bigger

Smaller

$\lim_{x\to\infty} e^x > \lim_{x\to\infty} x^{400}$

d) $\lim_{x\to\infty} e^{-x}\cos x = 0$

$\lim_{x\to\infty} \frac{1}{e^x}\cdot\cos x = \lim_{x\to\infty} \frac{\cos x}{e^x}$

$0 = \frac{1}{RBN}$ or $\frac{-1}{RBN} = 0$

Biggest Smallest

e) $\lim_{x\to\infty} \left(\frac{3}{x}+2\right)\left(\frac{4x^2-1}{x^2}\right)$

$\left(\frac{3}{RBN}+2\right)\left(\frac{4x^2}{x^2}\right) = (0+2)(4) = (2)(4) = 8$

$\left(\frac{3}{x}+2\right)\left(\frac{4x^2-1}{x^2}\right) = \frac{3}{x}\cdot\frac{(4x^2-1)}{x^2} + \frac{2}{1}\cdot\frac{4x^2-1}{x^2} = \frac{12x^2-3}{x^3} + \frac{8x^2-2}{x^2}$

e) $f(x) = \frac{\sqrt{x^2+2}}{x-5} =$

$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{x-5} = \lim_{x\to\infty} \frac{\sqrt{x^2}}{x} = \lim_{x\to\infty} \frac{|x|}{x} = 1$

$\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{x-5} = \lim_{x\to-\infty} \frac{|x|}{x} = -1$

d) $f(x) = \frac{|x|}{x}$

$\lim_{x\to\infty} \frac{|x|}{x} = \frac{|50{,}000{,}000{,}000|}{50{,}000{,}000{,}000} = 1$

$\lim_{x\to-\infty} \frac{|x|}{x} = \frac{|-50{,}000{,}000{,}000|}{-50{,}000{,}000{,}000} =$

$\frac{50{,}000{,}000{,}000}{-50{,}000{,}000{,}000} = -1$

4. Find all the horizontal asymptotes of the function $y = \frac{8+2^x}{2+2^x}$

$\lim_{x\to\infty} \frac{8+2^x}{2+2^x} = \lim_{x\to\infty} \frac{2^x}{2^x} = 1$

$\lim_{x\to-\infty} \frac{8+2^x}{2+2^x} = \frac{8+0}{2+0} = \frac{8}{2} = 4$

$2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024}$

$2^{-12} = \frac{1}{2^{12}} = \frac{1}{4096}$

$2^{-100} = \frac{1}{2^{100}} =$

5. Sketch a function that satisfies the stated conditions. Include any asymptotes.

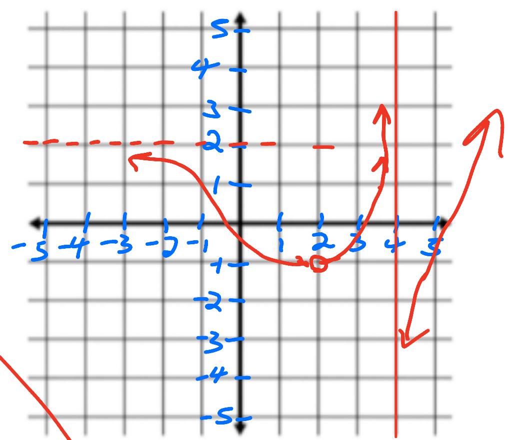
$\lim_{x\to 2} f(x) = -1$

$\lim_{x\to 4^-} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = 2$

$\lim_{x\to 4^+} f(x) = -\infty$

$\lim_{x\to \infty} f(x) = \infty$



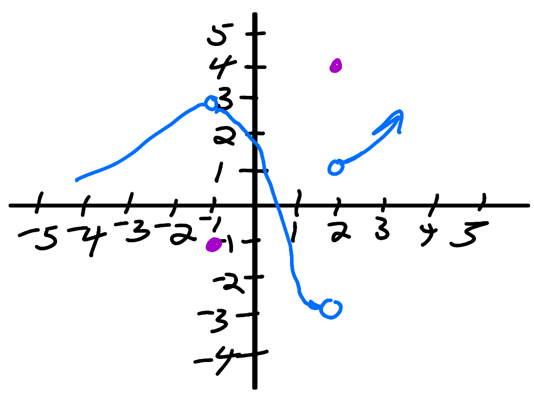

$F(-1) = -1$

$F(2) = 4$

$\lim_{x\to 2^-} F(x) = -3$

$\lim_{x\to 2^+} F(x) = 1$

$\lim_{x\to -1} F(x) = 3$




$$F(x) = \begin{cases} 2x^2+3 & x < 2 \\ 11 & x = 2 \\ 5x+3 & x > 2 \end{cases}$$

$\lim_{x\to 2^+} F(x) = \lim_{x\to 2^+} 5x+3 = 5\cdot 2+3 = 13$

are not the same

$\lim_{x\to 2^-} F(x) = \lim_{x\to 2^-} 2x^2+3 = 2\cdot 2^2+3 = 11$

$8+3$

$\lim_{x\to 2} F(x) = \varnothing = \text{DNE}$

$$F(x)=\begin{cases} 7x+1 & x<2 \\ 15 & x=2 \\ x^3+2x & x>2 \end{cases}$$

$\lim_{x\to 2} F(x) = \emptyset$

$\lim_{x\to 2^-} F(x) = \lim_{x\to 2^-}(7x+1) = 7\cdot 2+1 = 15$

NOT THE SAME

$\lim_{x\to 2^+} F(x) = \lim_{x\to 2^+}(x^3+2x) = 2^3+2\cdot 2 = 12$

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$\lim_{x\to c} F(x)=9$ $\quad \lim_{x\to c} g(x) = -5$

$$\lim_{x\to c}\left[\tfrac{1}{3}F(x)+g(x)\right] = \tfrac{1}{3}\cdot 9 + {-5} = 3-5 = -2$$

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$\lim_{x\to c} F(x) = -8$ $\quad \lim_{x\to c} g(x) = -7$

$$\lim_{x\to c}\left[3F(x) - g^2(x)\right] = 3\cdot(-8) - (-7)^2$$

$$-24 - 49 = -73$$

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$F(-3) = \emptyset$

$\lim_{x\to -3} F(x) = -2$

$F(-1) = 0$

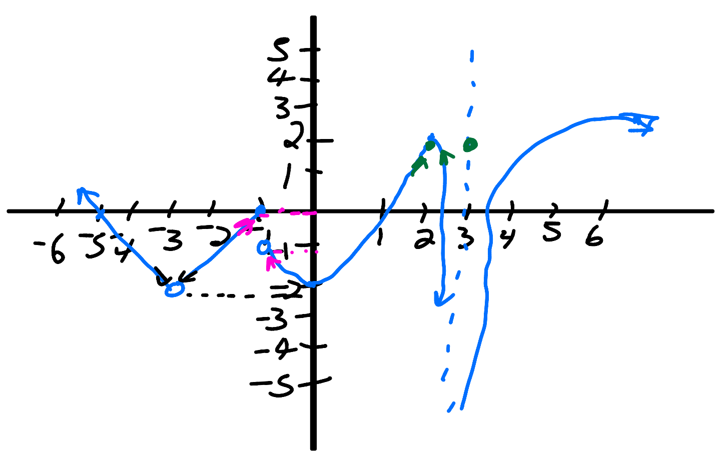
$\lim_{x\to -1} F(x) = \emptyset$

$F(2) = 2$

$\lim_{x\to 2} F(x) = 2$

$F(3) = 2$

$\lim_{x\to 3} F(x) = \text{DNE} = -\infty$



$$\lim_{x\to 1} \cos\frac{\pi x}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\lim_{x\to 7} \frac{2x-14}{x^2-49} = \frac{2\cdot 7-14}{7^2-49} = \frac{14-14}{49-49} = \frac{0}{0} = \varnothing$$

$$\lim_{x\to 7} \frac{2\cancel{(x-7)}}{(x+7)\cancel{(x-7)}} = \frac{2}{7+7} = \frac{2}{14} = \frac{1}{7}$$

$$\lim_{x\to 5} \frac{2x-10}{x^2-25} = \lim_{x\to 5} \frac{2\cancel{(x-5)}}{\cancel{(x-5)}(x+5)} = \frac{2}{5+5} = \frac{2}{10} = \frac{1}{5}$$

$$\frac{\sqrt{4+12}-4}{4-4}$$

$$\lim_{x\to 4} \frac{(\sqrt{x+12}-4)}{(x-4)} \frac{(\sqrt{x+12}+4)}{(\sqrt{x+12}+4)}$$

$$\frac{\sqrt{16}-4}{4-4} = \frac{4-4}{4-4} = \frac{0}{0}$$

$$\frac{x+12+\cancel{4\sqrt{x+12}}-\cancel{4\sqrt{x+12}}-16}{(x-4)(\sqrt{x+12}+4)} = \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x+12}+4)}$$

$$\frac{1}{\sqrt{4+12}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \frac{1}{8}$$

$$\lim_{x\to 2} \frac{(\sqrt{x+7}-3)}{(x-2)} \frac{(\sqrt{x+7}+3)}{(\sqrt{x+7}+3)} = \lim_{x\to 2} \frac{x+7+\cancel{3\sqrt{x+7}}-\cancel{3\sqrt{x+7}}-9}{(x-2)(\sqrt{x+7}+3)}$$

$$\frac{\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x+7}+3)}$$

$$\lim_{x\to 2} \frac{1}{\sqrt{x+7}+3} = \frac{1}{\sqrt{2+7}+3}$$

$$\frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

$$\lim_{x\to 0} \frac{2}{2}\cdot\frac{\sin 2x}{5x} = \lim_{x\to 0} \frac{2\sin 2x}{5\cdot 2x} = \frac{2}{5}\cdot 1 = \frac{2}{5}$$

$$\lim_{x\to 0} \frac{5}{5}\cdot\frac{\sin 5x}{2x} = \lim_{x\to 0} \frac{5\sin 5x}{2\cdot 5x} = \frac{5}{2}\cdot 1 = \frac{5}{2}$$

$\sin 5x \neq 5\sin x$

$\sin 5\cdot\frac{\pi}{2} \neq 5\cdot\sin\frac{\pi}{2}$

$\sin 2\frac{1}{2}\cdot\pi \neq 5\cdot 1$

$1 \neq 5$

$$\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$$

$$\frac{4}{4}\cdot\frac{1}{(x+4)} - \frac{1(x+4)}{4(x+4)}$$

$$\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)} = \frac{\cancel{4} - x \cancel{- 4}}{4(x+4)}$$

$$\lim_{x\to 0} \frac{\frac{-x}{4(x+4)}}{\frac{x}{1}} = \lim_{x\to 0} \frac{-\cancel{x}}{4(x+4)}\cdot\frac{1}{\cancel{x}}$$

$$\lim_{x\to 0} \frac{-1}{4(x+4)} = \frac{-1}{4(0+4)}$$

$$\frac{-1}{4\cdot 4} = \frac{-1}{16}$$

$$\lim_{x\to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$$

$$\frac{5}{5}\cdot\frac{1}{(x+5)} - \frac{1(x+5)}{5(x+5)}$$

$$\frac{5}{5(x+5)} - \frac{x+5}{5(x+5)} = \frac{\cancel{5} - x \cancel{- 5}}{5(x+5)}$$

$$\lim_{x\to 0} \frac{\frac{-x}{5(x+5)}}{\frac{x}{1}}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{5(x+5)}\cdot\frac{1}{\cancel{x}} = \lim_{x\to 0} \frac{-1}{5(x+5)} = \frac{-1}{5(0+5)} = \frac{-1}{5\cdot 5} = \frac{-1}{25}$$

$$-x^2 + 4x - 1 \le F(x) \le x^2 - 4x + 7$$

$$3 \le \lim_{x\to 2} F(x) \le 3$$

Squeeze Theorem

$\lim_{x\to 2} F(x)$

$\lim_{x\to 2} -x^2 + 4x - 1 = -(2)^2 + 4(2) - 1$

$-4 + 8 - 1 = 3$

$\lim_{x\to 2} x^2 - 4x + 7 = 2^2 - 4\cdot 2 + 7$

$4 - 8 + 7 = 3$

$$\lim_{x\to s} \frac{x-s}{\sqrt{x}-\sqrt{s}} =$$

$$\frac{(\sqrt{x})^2-(\sqrt{s})^2}{\sqrt{x}-\sqrt{s}} = \frac{(\sqrt{x}-\sqrt{s})(\sqrt{x}+\sqrt{s})}{\cancel{(\sqrt{x}-\sqrt{s})}}$$

$$\frac{(x-s)(\sqrt{x}+\sqrt{s})}{(\sqrt{x}-\sqrt{s})(\sqrt{x}+\sqrt{s})} = \frac{(x-s)(\sqrt{x}+\sqrt{s})}{x+\cancel{\sqrt{s}x}-\cancel{\sqrt{s}x}-s} = \frac{\cancel{(x-s)}(\sqrt{x}+\sqrt{s})}{\cancel{(x-s)}}$$

$$\lim_{x\to s} \sqrt{x}+\sqrt{s} = \sqrt{s}+\sqrt{s} = 2\sqrt{s}$$